LAB REPORT: LAB 5

TNM079, MODELING AND ANIMATION

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Abstract

This report describes the theory and implementation for a level set framework. Morphological operations like erosion, dilation and the combinations of these are implemented. And the surface is advected in a vector field using upwind scheme. The result of the implemented level set framework is presented and discussed.

1 Introduction

A level set is a subset of an implicit surface which can be deformed by partial differential equations (PDEs). These are often used in computer graphics. The hyperbolic class of PDEs is used in this report.

2 Background

A level set is defined by the level set function ϕ as following

$$S = \{x \in^d : \phi(x) = h\} \tag{1}$$

where h is the isovalue. Setting h to zero can be done for easy sign convention. This defines the points inside and outside of S as

$$S_{inside} = \{x \in^d : \phi(x) < 0\} \tag{2}$$
$$S_{outside} = \{x \in^d : \phi(x) > 0\} \tag{3}$$

It is known that for isolines and isosurfaces the normal of any level set is

$$n = \frac{\nabla\phi}{|\nabla\phi|} \tag{4}$$
$$\tag{5}$$

To be able to deform a surface over time, time dependency needs to be introduced. One way of doing this is to make the level set a function of time in equation 1. To move the level set, a point on the level set α(t) is on the surface for all t. Differentiating with respect to time gives

$$\frac{\delta\phi}{\delta t} = -\nabla\phi \cdot \frac{d\alpha}{dt} \tag{6}$$
$$= -F|\nabla\phi| \tag{7}$$

where F is a level set speed function that describes the speed of α in the normal direction. F can be written as

$$F = \widehat{n} \cdot \frac{d\alpha}{dt} = \frac{\nabla\phi}{|\nabla\phi|} \cdot \frac{d\alpha}{dt} \tag{8}$$

To be able to solve the function in the computer it needs to be made discrete. This is done in this case by using a forward Euler scheme for its simplicity:

$$\frac{\delta\phi}{\delta t} \approx \frac{\phi^{n+1} - \phi^n}{\Delta t} \tag{9}$$

The spatial discretization relies strongly on what PDE is used and in this report hyperbolic is used. To go in to hyperbolic advection one first need to understand advection. Advection is to move the level set surface. Hyperbolic advection is to advect in a vector field or a normal direction. The vector field can represent physical forces as wind or fluids or twist and skews. In this report the vector field describe a kind of swirl. In contrary to equation 7 where we move in the normal direction, we can describe it as

$$\frac{\delta\phi}{\delta t} = -V \cdot \nabla\phi \tag{10}$$

where V is the vector field. Since we know the direction of the flow with the vector field V, upwind scheme can be used, here along the x-axis:

$$\frac{\delta\phi}{\delta x} \approx \begin{cases} \phi_x^+ = (\phi_{i+1,j,k} - \phi_{i,j,k})/\Delta x, & V_x < 0 \\ \phi_x^- = (\phi_{i,j,k} - \phi_{i-1,j,k})/\Delta x, & V_x > 0 \end{cases} \tag{11}$$

where $\phi_{i,j,k}$ is a discrete sample point at position (i,j,k) in the grid and Δx is the grid spacing. When advecting in the direction of the surface normal the direction of the flow is not known. For this Godunov's method can be used:

$$(\frac{\delta\phi}{\delta x})^2 \approx \begin{cases} max[max(\phi_x^-, 0)^2, min(\phi_x^+, 0)^2], & F > 0 \\ max[min(\phi_x^-, 0)^2, max(\phi_x^+, 0)^2], & F < 0 \end{cases} \tag{12}$$

The Courant-Friedrichs-Lewy (CFL) stability condition states that only the sample points behind, or up-wind to, the wave should be used in the discretization since these are the only points with reliable information. To ensure stability the timestep can not be allowed to be greater than the wave speed given by V or F resulting in:

$$\Delta t < min\{\frac{\Delta x}{V_x}, \frac{\Delta y}{V_y}, \frac{\Delta z}{V_z}\} \tag{13}$$

$$\Delta t < \frac{min\{\Delta x, \Delta y, \Delta z\}}{|F|} \tag{14}$$

An important process to keep the surface stable is *reinitialization* which should be performed frequently. Reinitialization is used to satisfy the Eikonal equation:

$$|\nabla\phi| = 1 \tag{15}$$

This implies that the level set function is properly reinitialized when its gradients have the same length.

3 Tasks

Implementation of a level set framework is explained in this section.

3.1 Implementing differential calculations

In the level set framework following differential calculations where calculated along the x-axis, y-axis and z-axis but is only shown for the x-axis. The differential equations in equation 11 was implemented. Some differential equations where implemented but not used since implementation of mean curvature flow was not performed. These where the second order central difference schemes for parabolic diffusion [1].

3.2 Implementing erosion and dilation

To implement eurosion and dilation equation 7 was used. Since the direction of the flow is not known explicitly the Godunov method in equation 12 was used to calculate $\nabla\phi$. It resulted in the wave speed given by F. To get a stable time step, equation 14 was used, where the time step was multiplied with a number smaller than one to make sure it was always less than the wave speed given by F. Dilation (advection outwards of the surface) or erousion(advection inwards of the surface) was then decided by the sign of F.

3.3 Implementing advection from an external vector field

Implementing advection from an external vector field was done by using the equation 10 together with upwind scheme 11. Meaning if $V_x < 0$ then values from the positive side ϕ^+ is used since these have been visited. The opposite is true for $V_x > 0$. This was implemented for x,y and z. Then equation 10 was used to calculate the wave speed given by V. To get a stable time step, equation 13 was used, where the time step was multiplied with a number smaller than one.

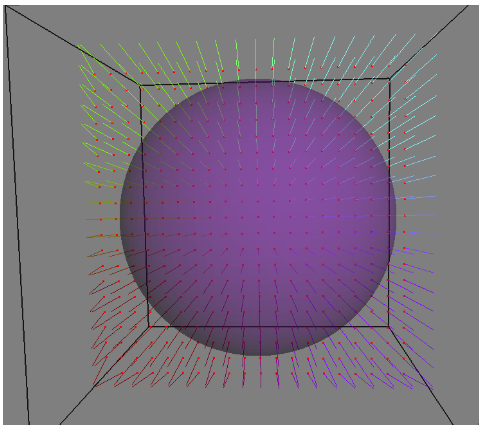


Figure 1: A levelset object with gradients

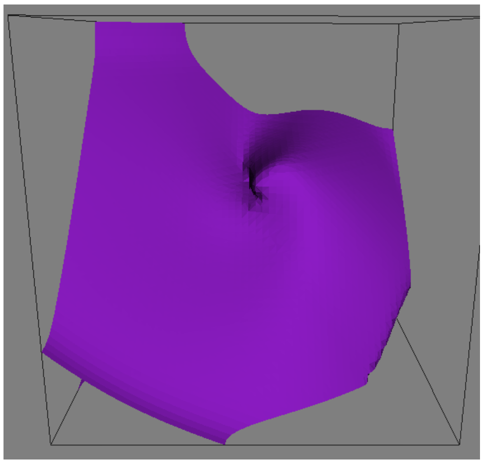


Figure 2: A levelset object advected in a vector field

4 Results

In figure 1 the gradients by using the difference scheme in equation 11 can be seen. This results in the center most gradient in an implicit sphere to be zero. When reinitializing the level set object, an implicit sphere, the gradients begun changing to the same size. This is seen by the scalar cutting plane in figure 3 by the scale of color change. However, the obtained result is not entirely correct since it is uneven at the edges and in the middle.The cause of this error is hard to point out.

The result of dilation and erosion is visualised in figure 4. It is noticeable that the lose bits of the horse-models leg can be removed both by dilation and erosion. Dilation provides a better result in this case. By using advection in a vector field an level set object, a plane, could change apperance. By using such a simple shape the vector field could be observed as a swirl which can be seen in figure 2.

5 Conclusion

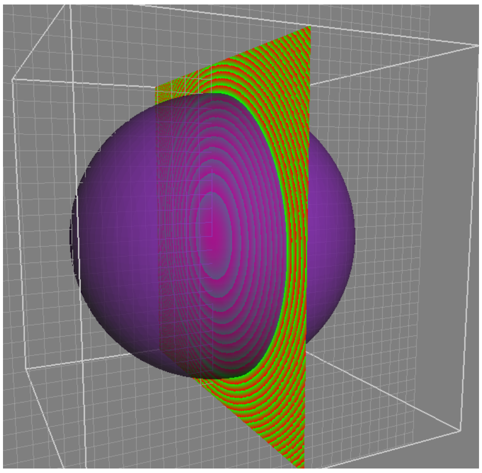
The level set representation is a little hard to wrap your head around but provides powerful ways to deform surfaces. The erosion and dilation operators work well together were they can be used in series to achieve morphological opening or closing.

6 Lab partner and grade

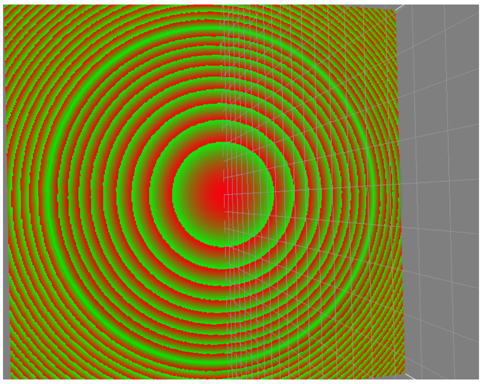
The lab was done together with Tim Olsson and aims for grade 3.

References

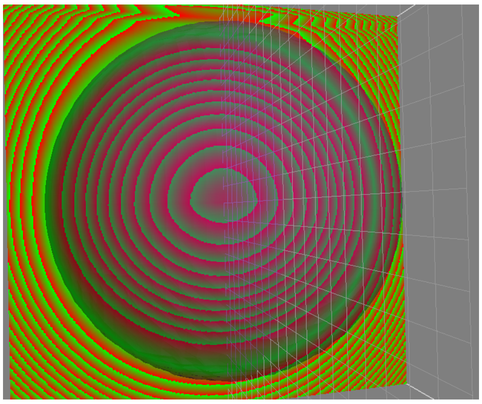
[1] Robin Skånberg Mark E. Dieckmann and Emma Broman. "level-set methods". 2021.



(a)

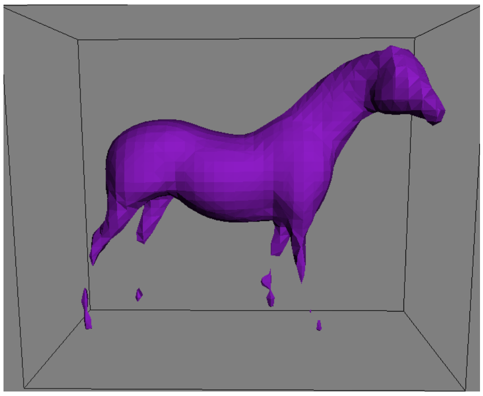


(b)

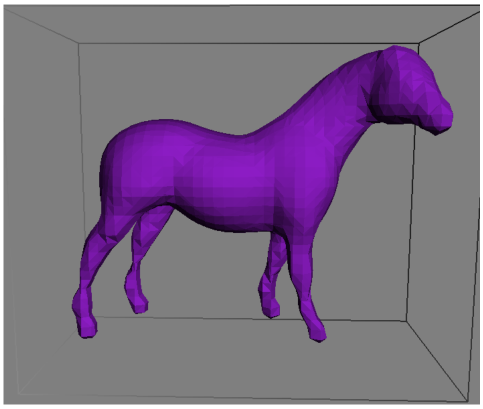


(c)

Figure 3: Result from task two showing (a) the level set object with a scalar cut plane, (b) the scalar cut plane without reinitialization, (b) the scalar cut plane that has been reinitialized 10 times



(a)



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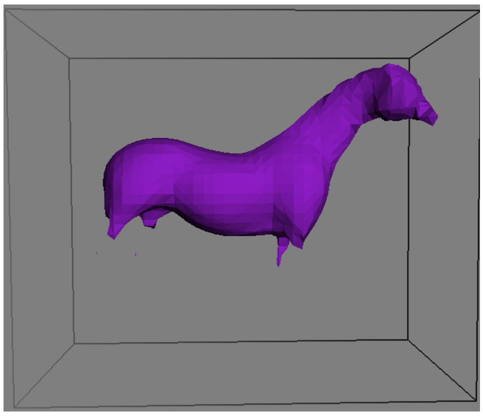

(c)

Figure 4: Result from task three showing (a) the original model, (b) the model after dilation, (b) the model after erosion