LAB REPORT: LAB 4

TNM079, MODELING AND ANIMATION

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Abstract

This report describes the theory and implementation of a modeling framework for implicit geometry. Especially constructive solid geometry such as union and intersection will be covered. Also quadric surfaces will be implemented. The result of the implementation is presented and discussed at the end of the report.

1 Introduction

Implicit surfaces are surfaces that are implicitly described by an equation. This makes implicit surfaces good to work with in modeling and animation since they are physically realistic. They are also often more compact than their explicit representation. In the report the concept constructive solid geometry hereby refereed to as CSG is used. This concept is based on the idea of creating more complex models by combining simpler objects by using union, intersection and difference.

2 Background

An implicit surface is defined as

$$S(C) \equiv \Big\{ \{x\} : f(x) = C \Big\} \tag{1}$$

In this lab we look at the zero level-set where C is zero. This gives that x can be classified as

$$\begin{cases} f(x) < 0, & \text{x is inside the surface} \\ f(x) = 0, & \text{x is on the surface} \\ f(x) > 0, & \text{x is outside the surface} \end{cases} \tag{2}$$

By testing all the points with this condition the surface can be built. Some attributes that are important when doing shading is the the surface normal and surface mean curvature. The normal of an implicit surface can be found through the gradient and assuming one of the basis vectors points in the same direction as the normal, it can be written as

$$\nabla f = e_1^{\rightarrow}(e_1^{\rightarrow} \cdot \nabla f) + e_2^{\rightarrow}(e_2^{\rightarrow} \cdot \nabla f) + n^{\rightarrow}(n^{\rightarrow} \cdot \nabla f) \tag{3}$$

where e_1, e_2 are vectors with lenght one and n is the normal. In the lab only manifold meshes was used, making it able to make the assumption that the surface contains infinity small flat patches which leads to that f will be locally constant everywhere. This gives the normal

$$n^{\rightarrow} = \pm \frac{\nabla f}{|\nabla f|} \tag{4}$$

But by using the sign convention in equation 2 we know that the normal can only be positive if it goes out from the surface.

A kind of surface is the quadric surface that is defined by a quadratic implicit function and can be written as

$$P^T Qp = \begin{pmatrix} x & y & z & 1 \end{pmatrix} \begin{pmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{5}$$

in matrix form for a point p. By using this general function a total of 17 different standard surfaces as planes, cylinders, cones, paraboids and more can be described.

The normal of an implicit is defined in equation 3. The gradient for a quadric surface is defined as

$$\nabla f(x,y,z) = 2 \begin{pmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{6}$$

By using implicit surfaces one can, as mentioned, build more complex models by combining simpler ones. The CSG object are constructed by using boolean operations on two or more objects. The boolean operations in this report between two implicit models A and B are

$$\begin{cases} Union(A,B) = A \cup B = min(A,B) \\ Intersection(A,B) = A \cap B = max(A,B) \\ Difference(A,B) = A - B = max(A,-B) \end{cases} \tag{7}$$

3 Tasks

Implementation of implicit geometry is explained in this section.

3.1 Implement CSG operators

The boolean operations presented in equation 7 was computed to create the CSG objects. By taking the minimum value at each grid point the positive values of one surface was overwritten if the other surface had negative values. From equation 2 we know that a negative value means that it is inside the surface. This joins the surfaces creating a union operator. Meanwhile by taking the maximum value at every grid point the result will only be negative, and inside the surface, where both surfaces are negative. Creating the intersection operator. The difference operator was computed by taking the maximum value at each grid point, with the inverted sign for one of the surfaces. This means that all grid points on that surface that was inside (negative value) is now outside (positive value). The result will only be negative where the surface with inverted sign crosses into the other surface. These cases can be seen in figure 1

3.2 Implement the quadric surface

The different standard surfaces included in the task was

$$\begin{cases} Plane: f(x,y,z) = ax + by + cz = 0 \\ Cylinder: f(x,y,z) = x^2 + y^2 - 1 = 0 \\ Ellipsoid: f(x,y,z) = x^2 + y^2 - 1 = 0 \\ Cone: f(x,y,z) = x^2 + y^2 - z^2 = 0 \\ Parabloid: f(x,y,z) = x^2 \pm y^2 - z = 0 \\ Hyperboloid: f(x,y,z) = x^2 + y^2 \\ -z^2 \pm 1 = 0 \end{cases} \tag{8}$$

By using the general function 5 and adjusting Q, all the standard surfaces above could be represented. For example was a cone constructed by adjusting Q in the following way

$$P^T Qp = \begin{pmatrix} x & y & z & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{9}$$

And a two sheet hyperboloid was constructed by

$$P^T Qp = \begin{pmatrix} x & y & z & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{10}$$

according to their respective equations in equation 8. The surface gradients where computed by equation 6.

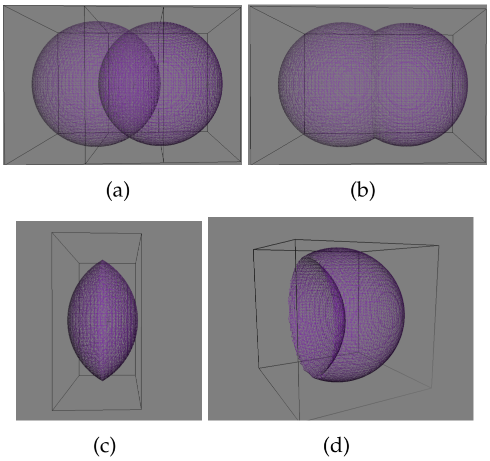

Figure 1: Result from task one showing (a) the original models, (b) the union, (c) the intersection, (d) the difference

4 Results

The result of implementing the CSG operations and applying these to two implicit spheres ca be seen in figure 1.

The result of implementing the quadric implicit surface can be seen in figure 2 by six different standard implicit surfaces created with the same method. The gradients of an implicit quadric surface can be viewed in figure 3. The gradients where parallel with the normals as expected.

5 Conclusion

The CSG operations provide a simple way to make interesting models from simpler models. A drawback is however that sharp and ill defined edges are created, as seen quite well in all figures in 1. No normals are defined where the surfaces intersect, this is why especially the figure 1(d) has a highly uneven edge.

As seen, quadric implicit surfaces can represent a wide variety of different standard surfaces. Combining these standard surfaces with the CSG operators, more interesting shapes can be represented. The implementation of the gradients for a quadric surface was simple since it involves the same coefficents as the quadric function.

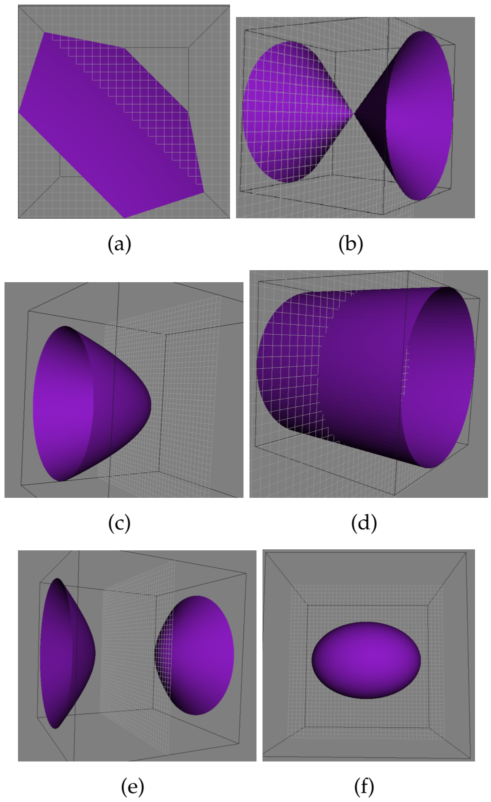

Figure 2: Result from task two showing (a) an implicit plane, (b) an implicit cone, (c) an implicit paraboloid, (d) an implicit cylinder, (e) an implicit two sheet hyperboloid, (f) an implicit ellipsoid

6 Lab partner and grade

The lab was done together with Tim Olsson and aims for grade 3.

References

[1] Robin Skånberg Mark E. Dieckmann and Emma Broman. “implicit surfaces and modeling”. 2021.

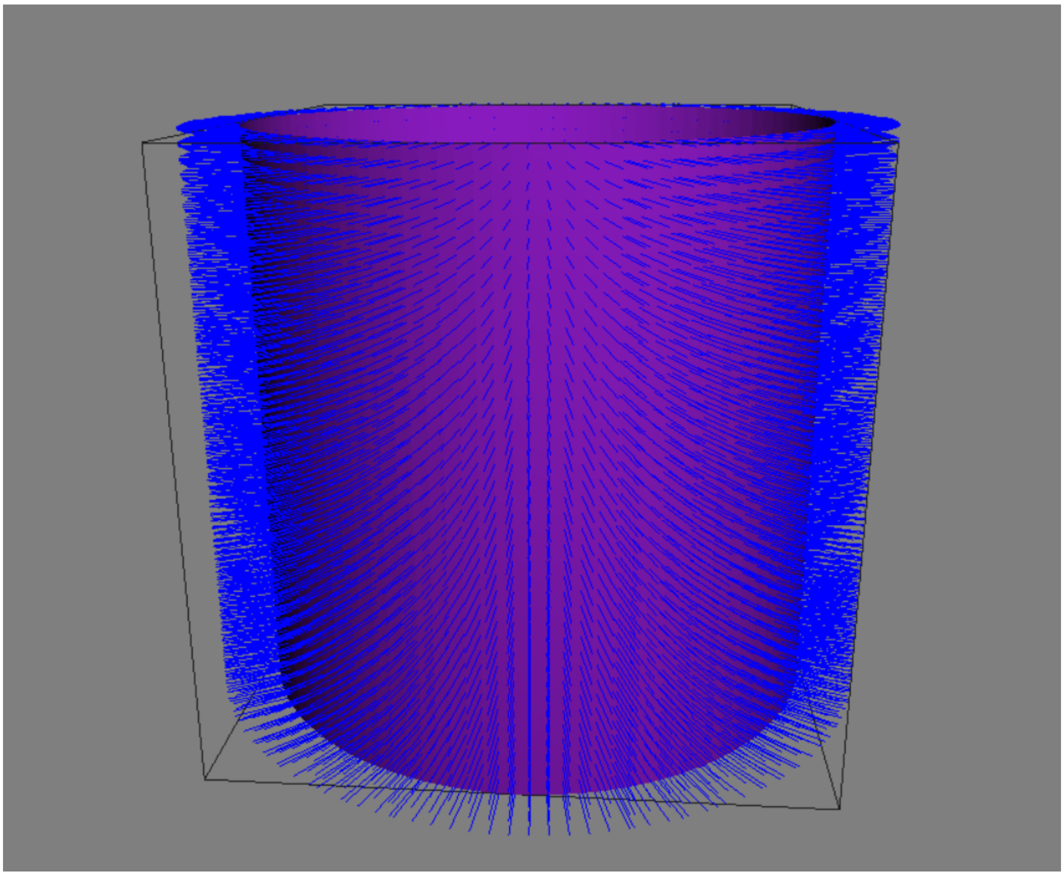

Figure 3: Gradients for an implicit surface