LAB REPORT: LAB 2

TNM079, MODELING AND ANIMATION

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Abstract

This report describes the implementation of mesh decimation which is often used in computer graphics to simplify complex models. Mesh decimation using quadric error metrics is discussed and implemented, the result from using the algorithm is visualized and artifacts are inspected.

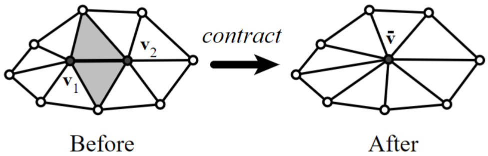

Figure 1: Edge contraction, from [1]

1 Introduction

An essential part of computer graphics is to find a balance between complex models and computational cost. Mesh decimation is used to reduce the level of detail of models whenever possible. It is important that the model keep its distinct shape after the decimation and therefore that the new vertex points are calculated correctly. By using quadric error metrics as a cost evaluation for the vertex points this can be achieved.

2 Background

In this lab a decimation algorithm was implemented where the focus was on the error metrics used as a cost for choosing the order in which the edges where contracted. Edges where then collapsed repeatedly until the requested mesh reduction was achieved. Vertices in the mesh was divided into pairs *v1* and *v2* as seen in figure 1 and one contraction would, as seen in the figure, move the vertices to a new position $\bar{v}$. The optimal new vertex position was found by computing an error quadric matrix for each vertex.The quadric matrix was computed from the distance to the planes connected to the vertex which is further explained in section 3.1 . It is necessary to have a good algorithm for decimation to be able to contain the shape of the mesh. The decimation algorithm used in this lab is introduced in *Surface Simplification Using Quadric Error Metrics* by Garland and Heckbert [1]. The approach uses quadric error metric to compute the optimal contraction target. The algorithm contains both edge contraction and none edge contraction, see figure 1 and 2. However in this lab only edge contraction is implemented and discussed for simplicity. This implies that the algorithm implemented in this lab is compatible with manifold surfaces only. Some main advantages of the algorithm is that it is efficent and creates high quality approximations. Hovewer the main advantage that Garland and Heckbert's algorithm has is that it can join unconnected regions.

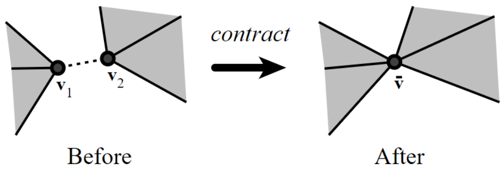

Figure 2: Non-edge contraction, from [1]

3 Tasks

The mesh decimation was by done by contracting edges. The contraction was executed by moving the vertices *v1* and *v2* to a new vertex position $\bar{v}$. This is called pair contraction. There is several algorithms that does pair contraction, the difference between these algorithms is how the contraction target $\bar{v}$ is picked. In this case the contraction target was picked by using the approximated error for $\bar{v}$ as a cost for each contraction. The contraction target with the lowest cost was then chosen.

3.1 Computing the error quadric matrices

Every vertex v in the mesh is connected to a set of faces that each have a plane p. The planes are defined by the equation

$$ax + by + cz + d = 0 \tag{1}$$

Where a, b, c is the normal vector N of the face and

$$d = ax + by + cz = N \cdot v \tag{2}$$

By taking the sum of the squared distances, between the planes that intersects at v and a moved vertex $\bar{v}$, an error metric where computed. The error metric in quadratic form is defined by using the fundamental error quadric for each plane p.

$$K_p = pp^T = \begin{bmatrix} a^2 & ab & ac & ad \\ ba & b^2 & bc & bd \\ ca & cb & c^2 & cd \\ da & db & dc & d^2 \end{bmatrix} \tag{3}$$

The error quadric Q is then defined by the sum of Kp for the planes intersecting v. An error quadric Q was defined for each vertex in the mesh.

3.2 Computing the optimal contraction target

The error metric in quadratic form for each vertex is defined as.

$$\Delta v = v^T Q v \tag{4}$$

To get the error quadric for $\bar{v}$, an approximated matrix $\bar{Q}$ was created by adding together $Q1$ and $Q2$, the error quadrics for the pair used in the pair contraction. To find the optimal position for $\bar{v}$, the lowest cost should be considered, meaning we want the lowest Δv. By deriving Δv in x,y and z its minimum value could be found. In matrix form this is defined as

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{23} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \bar{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

and if the matrix is invertible, meaning it has a non zero determinant, $\bar{v}$ could be found by the following equation.

$$\bar{v} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{23} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

If the matrix was not invertible the position $\bar{v}$ was estimated by either choosing the position of one of the vertices in the pair or the position in the middle of them. The method producing the lowest cost was then chosen.

4 Results

The resulting algorithm could be compared with a simple error metric where the new position was calculated to be half-way along an

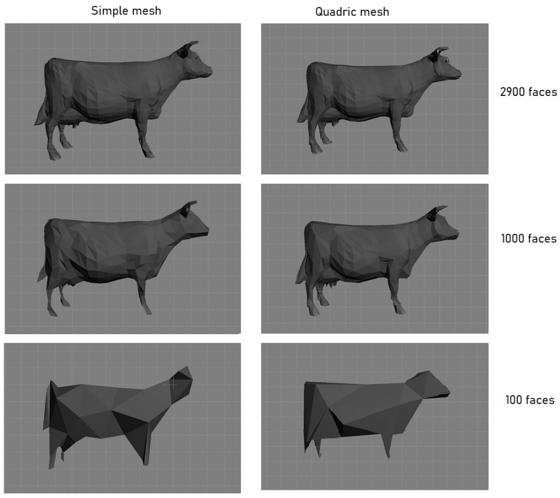

Figure 3: Result showing the difference of quality between using a simple error metric and quadric error metric for different number of faces.

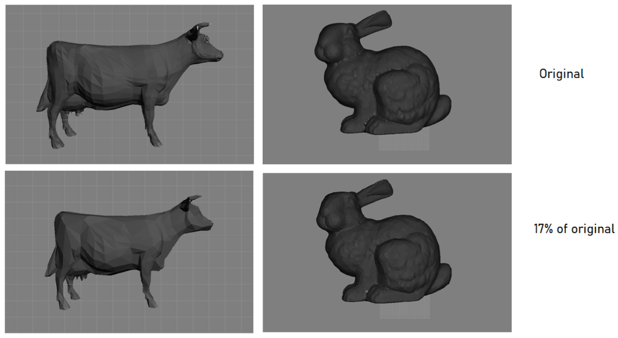

Figure 4: Result showing the difference between two models that are decimated to the same percent of the original mesh. Where as the mesh to the right originally has 5804 faces and the mesh to the left has 35032 faces. The decimation gives the mesh to the right 987 faces and the mesh to the left 5955 faces.

edge and the error was the distance to this new position. The difference in how the contraction target was picked made a great difference as can be seen in figure 3. It can be seen that for the same number of faces the mesh made with the quadric error metrics kept its structure a lot more than the mesh made with the simple error metrics.

Figure 4 shows the difference of quality by performing decimation to the same percent for two different meshes with different number of faces originally.

5 Conclusion

By looking at the result from this lab it can be seen that the quadric error metric is useful and provides good results. It can be seen in figure 3 that by using the quadric error metrics the overall shape of the cow was better preserved even for low numbers of faces. Where as the head was deformed in the simple error metric mesh and it overall performed a more uniform decimation. The quadric error metric preserves more triangles in more detailed areas which can be seen in figure 3 for 100 faces, where the mesh has fewer faces on the body than the simple mesh. This shows how, by using the quadric error metric, the algorithm performs a better decisions in which edge to be collapsed and improves the look of the mesh. It can further be seen in figure 4 that the quadric error metric works well for different detailed meshes, where a mesh who orginally has more faces also preserves more detail when decimated the same percent.

6 Lab partner and grade

The lab was done together with Tim Olsson and aims for grade 3.

References

[1] Michael Garland and Paul S. Heckbert. *"Surface simplification using quadric error metrics*. Carnegie Mellon University, 1997.

[2] Robin Skånberg Mark E. Dieckmann and Emma Broman. "mesh decimation". 2021.